The Aesthetics of Structures

adapted from the 2016 CUREE Calendar
illustrated essays by Robert Reitherman

© 2015 CUREE - All Rights Reserved.
The Aesthetics of Structures
by Robert Reitherman, CUREE Executive Director

Two Latin phrases help to introduce the subject of the aesthetics of structures.

**utilitas, firmitas, and venustas**
The first is from Vitruvius, written about two thousand years ago:

*utilitas* (usefulness, functionality), *firmitas* (stability, structural integrity and strength), and *venustas* (beauty) (Vitruvius c. 15 BC, vol. I).

Vitruvius stated that successful architecture must have all three qualities -- usefulness, structural integrity, and beauty. Relevant here are these questions: can the structural aspect of a work of architecture or other construction make a positive contribution to its beauty? Can the structure be designed so that in and of itself it is beautiful? Are there structural principles that can make buildings and other structures more beautiful or aesthetically satisfying?

**de gustibus non est disputandum**
If the following Latin proverb is correct, aesthetic judgments are purely judgmental, a matter of individual taste rather than subject to general principles:

*de gustibus non est disputandum* (in matters of taste, there is no dispute).

Hugh Blair (1718-1800) cited that proverb and asked “Is there any thing that can be called a standard of Taste, by appealing to which we may distinguish between a good and a bad Taste?” (Blair 1783 p. 15) Blair argued the pro side of this debate, further asserting that one’s sense of taste could be improved, which in connection with the present topic is related to developing the necessary engineering and aesthetic sensibilities to judge whether a structure is beautiful. How can structural engineers produce structures that are beautiful in addition to being efficient with respect to their cost and social utility? Are there any general principles that often apply to successful instances of this type, even if, being a matter of taste, the principles can not be rigidly and universally applied?

Blair’s logic began by taking up the proposition of the proverb that all tastes, all opinions regarding aesthetics, are equally valid. He found that this position, “when we apply it to the extremes, presently shows its absurdity.” In the structural context, one might cite the original and final designs for the Golden Gate Bridge. See Figure 1 and Figure 2.

**Figure 1.** The original design by Joseph Strauss for the Golden Gate Bridge. *Golden Gate Bridge, Highway and Transportation District Archives.*

**Figure 2.** The eventual design used for the Golden Gate Bridge.
The combined cantilever-suspension bridge that was the original proposed design for the Bridge does have a structural rationale in shortening the span of the suspension structure. It is the same basic design that master structural designer T. Y. Lin later proposed for a heroic multi-span bridge across the Strait of Gibraltar, (though Lin’s was more spare and rational-looking). Most people who compare the two Golden Gate Bridge designs prefer the one that got built (and might say, “Whew! I’m sure glad they didn’t pick the other design, the one that looks like two bridges collided on a foggy night.”) Merely conforming to majority opinion is not necessarily a sound way to decide questions of taste or aesthetics, but it does bring up the point that for a structure to serve a public aesthetic purpose, it has to be appreciated, and the more widely it is appreciated, the greater the public good.

**Structural Elegance**
In considering these two designs for the Golden Gate Bridge, assuming one agrees that the eventual design is more beautiful, the question is why? Perhaps the simplicity of the all-suspension design appeals to us, as compared with the busier cantilever-suspension design. To an engineer, a simple and direct structural solution using a minimum of material is a virtue, and the original design was not as simple and had many more members. It was a combination of structural concepts that are harder to understand than the simple suspension bridge configuration. At a more detailed scale, one notes that the trussed deck for the suspended span is quite different than the thin girder deck of the side spans, which makes one wonder why. The deck of the final design stretches from end to end continuously drawing the eye across. (That is, the deck is continuous in visual terms -- in actual engineering terms it is three separate elements, two suspended side spans and the central span.) The central, flexible span of the first design for the bridge needed more bracing to resist wind effects, but the eye still sees a discontinuity that is not present in the case of the final design. Discontinuities, as compared with smooth transitions, are not necessarily always a negative visual trait, but they do keep the mind from following one theme through to completion. Especially for the non-engineer, if the logic behind a change in structural systems is not readily apparent, the viewer’s aesthetic experience is marginalized.

The term elegant is often used to describe a structure one finds to be beautiful, and elegance implies a simple and direct handling of the forces, a logic inherent in the load path, and this often leads to a structure that is easy to read. It also implies use of a minimum of material, which in turn can imply that the structure is not covered up or cluttered with superficial decoration. “Slender” more often than “stocky” is an adjective that describes structures that we call elegant, as in the slender columns of **Figure 3**.

![Figure 3 - The slenderness of the columns, accentuated by using pairs rather than single, larger-diameter columns, is part of the elegance of this example of architecture (the Casino, Avalon, Catalina Island, California).](image-url)
Sometimes, a member of very compact proportions can visually express a sense of strength (Figure 4) and have a positive aesthetic impact. And sometimes, in the author’s opinion, an overly stocky column with gross rather than refined detailing can have a negative aesthetic impact; see Figure 5. (If a column of such low slenderness ratio was needed for structural reasons, it could have been integrated with the horizontal structure more elegantly.)

Figure 4 - The tree-trunk column is obviously not slender, and it only holds up one end of a ridge beam of a small rest room building in Yosemite National park. It is not efficiently using a minimum of material. Nonetheless, it has an aesthetic appeal and is in proportion to the other stocky components of the structure.

Figure 5 - A column supporting an elevated highway in Dubai, United Arab Emirates, whose appearance would have been improved with more slender proportions and a capital of unified form and expressed structural purpose.

If one argues that columns that are more slender are always more appealing than stockier ones, then one must argue that Ionic and Corinthian columns are preferable to Doric ones -- and yet most architectural historians have nothing negative to say about the appearance of the Doric peristyle around the Parthenon. The classical orders were not simply defined by the types of capitals on the columns: the Doric column was proportioned to have a height to diameter ratio, slenderness ratio, of four to six (Fletcher 1975 p. 204); Ionic columns a slenderness ratio of nine (p. 223). A Corinthian column is usually about ten times taller than its diameter. See Figure 6 and Figure 7.

Structural Art
David Billington (1985, 1990, 2003) has introduced the term structural art to refer to works of structural engineers that are creative to the point of being
art. In his definition of the term, the art is created by an engineer, not by an architect nor via an architect-engineer collaboration. From that perspective, one can consider a structure as functional sculpture, the sculptural aspect being a matter of aesthetics but integrated with the functional or structural aspect created by the engineer.

Billington’s three criteria for the achievement of structural art are efficiency, economy, and elegance. Efficiency is a traditional concern of the structural engineer, for example, picking a steel wide flange beam of minimum weight to span a particular distance. Billington’s concept of economy includes minimizing the first cost of the structure (hence one reason for his disdain for the new East Span of the San Francisco-Oakland Bay Bridge -- a bridge that could have been built for one-fifth of its $7 billion cost.) It also includes life cycle costs, which in today’s jargon would be called sustainability. His term elegance is the aesthetic aspect, which in his definition of structural art is not superficial beautifying of a structure but a discovery by the engineer of a beautiful form that meets engineering requirements.

Using those criteria, the works of structural engineers such as Gustave Eiffel, Robert Maillart, Othmar Ammann, Christian Menn, Felix Candela, Pier Luigi Nervi, and Fazlur Khan are structural art.

Today there is a network of interested educators and researchers who comprise the International Network for Structural Art. Also underway is a project headed by Professor Maria Garlock at Princeton University, where Professor Emeritus Billington taught for many years, to broadly disseminate the resources and techniques faculty can use to teach similar courses that integrate the social and aesthetic aspects into traditional civil engineering courses on structures. That project, the Creative Art of Civil/Structural Engineering, is archiving resources such as lectures, homework assignments, reading lists, and other content that will be freely available.

**Architects and Engineers**

While Billington’s thesis concerning structural art is limited to works created solely by engineers, one can also point to examples of successful collaborations of architects and engineers, with both disciplines sharing in the credit for the creative aspects of the appearance of the construction.
When differentiating architecture from engineering, and architects from engineers, we are dealing with the history of the nineteenth and later centuries, for previously these roles were not clearly demarcated nor were the two professions well developed. When we talk about structural engineering today, we implicitly mean the body of knowledge that includes insights developed only since the last two or three hundred years, such as statics, strength of materials, dynamics, and methods of analysis that can subject a design to a theoretical test to verify the adequacy of its strength, ductility, stiffness, reliability, and other characteristics. One might call the ancient Romans who designed and built aqueducts “engineers,” or call the master masons who built Gothic churches “engineers.” In this discussion, however, “engineer” is used to describe the person educated in the quantitative engineering theory and methods that have by and large only been existence since the late 1700s. The brief chronology of the development of selected structural analysis methods (Table 1) indicates that for a structure to be engineered in the modern sense of the word, a knowledge base was necessary that was not in existence prior to approximately 1800. However impressive the constructions of the Egyptian, Persian, Greek, Roman, Romanesque, Gothic, and Renaissance eras, the only mathematics employed in their design was geometry, to make the pieces fit together. Forces and stresses were not yet calculable. Many more structural engineering developments in Table 1 could be cited, but they would be clustered in the later decades of the timespan shown there and would thus prove the point made here even more convincingly.

**Revealing Structural Behavior**

One former student of Billington’s, Michael Hein, states that “The elegant works of structural art are clear illustrations of structural behavior (2003, p. 127).” Here we have another potential criterion for judging the aesthetic value of a structure. With a little engineering background, one’s curiosity is piqued when looking at a structure -- how does it work? In many instances, the structural behavior of the members and connections is hidden, and for good reason. Buildings need wall-like elements to clothe their exterior and keep the elements out, often obscuring the structure inside. Bare steel loses half its strength when heated to only 1,000°F, 540°C, and building fires can easily surpass that temperature, often making it necessary to cloak the steel structure in fireproofing. The reinforcing steel in reinforced concrete clearly diagrams where the forces of tension or shear are flowing -- but concrete is opaque, not transparent. Nonetheless, many designers have been able to express the structure of their designs, making the construction “clear illustrations of structural behavior.”

Recognizing the behavior of the flying buttress, that it reaches over to where a masonry arch inside the building is exerting an outward lateral thrust, one finds it a convincing part of the architecture of a Gothic cathedral. When flying buttresses have multiple arms at different levels, they can delineate the multiple places inside the building from where the arches spring and where horizontal wind forces from the tall roofs would otherwise have to be resisted by thick walls (Borg and Mark 1973). Although the timber trussed gable roofs that always covered the stone vaults had tie beams, one theory is that if fire or rot destroyed the timber tie beam, the tension that was resisted by this chord could also have been resisted by the exterior buttress. See Figure 8.

**Figure 8** - Sections through flying buttresses of Gothic cathedrals, expressing the logic of resistance to the lateral thrust of arches spanning across the interior of the buildings and to horizontal wind forces from the roofs. (Arnold and Reitherman 1982 p. 187)
<table>
<thead>
<tr>
<th>Date</th>
<th>Development of Structural Engineering Analysis Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>c. 250 BC</td>
<td>Mathematical exposition of the lever or moment principle by Archimedes (c. 287-212 BC)</td>
</tr>
<tr>
<td>1585</td>
<td><em>De Thiende</em> (The Tenth) by Simon Stevin (1648-1620), popularized the Chinese and Arabic invention of decimals</td>
</tr>
<tr>
<td>1585</td>
<td><em>De Beghinselen der Weegconst</em> by Simon Stevin, resolution of forces</td>
</tr>
<tr>
<td>1638</td>
<td><em>Discourses and Mathematical Demonstrations Relating to Two New Sciences</em> by Galileo Galilei (1564-1642), principles of acceleration and inertia</td>
</tr>
<tr>
<td>1676</td>
<td>Hooke’s Law by Robert Hooke (1635-1703), law of elasticity</td>
</tr>
<tr>
<td>1687</td>
<td><em>Philosophiae Naturalis Principia Mathematica</em> by Isaac Newton (1643-1727), principles of inertia, forces, gravity</td>
</tr>
<tr>
<td>1705</td>
<td>Mathematical formulation of stress as force per unit area; curvature of a beam is proportional to its bending moment, Jakob Bernoulli (1654-1705)</td>
</tr>
<tr>
<td>1713</td>
<td><em>Essais et Recherces de Mathématique et de Physique</em> by Antoine Parent (1666-1716) internal force distribution in beams</td>
</tr>
<tr>
<td>1725</td>
<td>Modulus of elasticity, Leonhard Euler (1707-1783)</td>
</tr>
<tr>
<td>1757</td>
<td>Buckling of columns, Leonhard Euler</td>
</tr>
<tr>
<td>1773</td>
<td>Shear-related soil failures, internal force distribution in beams (not knowing of Parent’s earlier work), Charles-Augustin Coulomb (1736-1806)</td>
</tr>
<tr>
<td>1782</td>
<td>Experiments with modulus of elasticity, Giordano Riccati (1709-1790)</td>
</tr>
<tr>
<td>1800</td>
<td><em>Géométrie Descriptive</em> by Gaspard Monge (1746-1818), descriptive geometry, orthogonal/perspective drawing conventions</td>
</tr>
<tr>
<td>1807</td>
<td><em>A Course of Lectures on Natural Philosophy and the Mechanical Arts</em> by Thomas Young (1773-1829), Modulus of elasticity, E, Young’s Modulus, q.v. Riccati and Euler above; also relationship of vibration frequency to modulus of elasticity and inelastic absorption of kinetic energy</td>
</tr>
<tr>
<td>1823</td>
<td><em>Rapport et Mémoire sur les Ponts Suspendus</em> (Report on Suspension Bridges) by Claude Louis Marie Henri Navier (1785-1836)</td>
</tr>
<tr>
<td>1826</td>
<td><em>Résumé des Leçons Données à l’Ecole des Ponts et Chaussées</em> (Notes of Lectures Given at the School of Bridges and Roads) by Navier, plane sections remain plane, emphasis on calculation of stresses to have them remain elastic, rather than to calculate breaking loads; revised and corrected in the 1864 edition by Barré de Saint-Venant (1797-1886)</td>
</tr>
<tr>
<td>1838</td>
<td><em>Recherches sur la Probabilité des Jugements et en Matière Criminelle et Matière Civile</em> (Research on the Probability of Criminal and Civil Court Case Judgments) by Siméon Denis Poisson (1781-1840), Poisson distribution of random events; also Poisson’s ratio for the transverse strain in materials caused by longitudinal tension or compression</td>
</tr>
<tr>
<td>1847</td>
<td><em>An Essay on Bridge Building</em> by Squire Whipple (1804-1888), truss analysis by method of joints</td>
</tr>
<tr>
<td>1862</td>
<td><em>Die Graphische Statik</em> (Graphic Statics) by Wilhelm Ritter (1847-1906) and Karl Culmann (1821-1881)</td>
</tr>
<tr>
<td>1857</td>
<td>Emile Clapeyron (1799-1864), analysis of continuous beams</td>
</tr>
<tr>
<td>1864</td>
<td>James Clerk Maxwell (1831-1879), reciprocal theorem, analysis of statically indeterminate (moment-resisting) frames</td>
</tr>
<tr>
<td>1873</td>
<td><em>Intorno ai Sistemi Elastici</em> by Carlo Alberto Castigliano (1847-1884), relating forces, strain energy, and displacements (used for indeterminate structures)</td>
</tr>
<tr>
<td>1874</td>
<td>Otto Mohr (1835-1918), Maxwell-Mohr method for analysis of indeterminate structures</td>
</tr>
</tbody>
</table>

*source: (Reitherman 2012, p. 99)*
The outward thrusts of Roman and Romanesque arches were resisted by massive walls. To allow for the extensively glazed walls of a Gothic cathedral, the innovation of the flying buttress was sometimes necessary. (Flying buttresses were occasionally used in earlier styles, such as the Romanesque Durham Cathedral, which also had ribbed vaults). While originating as a structural innovation, most people today find the flying buttress on a Gothic building to be an aesthetic plus. Just imagine the outcry of historic preservationists and the public in general if the flying buttresses of Chartres, Notre Dame de Paris, or other Gothic cathedrals were proposed to be eliminated during a structural retrofit project. See Figures 9, 10, and 11.

Salvadori and Heller (1975) provide a commonsense explanation of the relationship of structure and beauty:

*It has been argued by some architectural historians, as well as by some structural engineers, that a deep concern for structure will unavoidably lead to beauty. It is undeniable that a ‘correct’ structure satisfies the eye of even the most unknowledgeable layman, and that a ‘wrong’ structure is often offensively ugly. But it would be hard to prove that esthetics are essentially dependent on structure. It is easy to show, instead, that some ‘incorrect’ structures are lovely, while some ‘correct’ ones are esthetically unsatisfying.*

---

**Figure 9** - St. Denis, Paris France, the first building to put all the elements of Gothic architecture together, including quite prominently the flying buttresses that flank its exterior.

**Figure 10** - Structural roles played by flying buttresses, Chartres Cathedral, Chartres, France. Masonry structures such as Gothic cathedrals can have complex behavior, even including shell behavior when analyzed by engineers today, but essentially the structural roles of the buttresses are clearly expressed and efficient given the materials and lack of structural analysis methods of the era.

**Figure 11** - A perspective section of Chartres cathedral showing the flying buttresses that are pictured in the photograph of Figure 10. Note that you are looking up at the structure as if it were lifted off its foundation. Not shown is the upper flying buttress, which was not part of the original design but was retrofitted on this side of the building during its prolonged construction. (Choisy 1899 p. 439)
Successful Architect-Engineer Collaborations
Are there examples of successful aesthetic structures designed by a combined team of architect and engineer? In the last half of the twentieth century, the Skidmore Owings and Merrill (SOM) firm became prominent in combining architecture and engineering “under one roof” (within one consulting company), as have the Arup and Foster practices that originated in the United Kingdom. Examples of striking architecture, and striking structure, such as the world’s tallest building, the Burj Khalifa, were products of a combined structural engineering + architecture design team at SOM, with the former discipline on that project headed by William Baker and the latter by George Efstathiou. While bridges can still be designed from start to finish almost completely by engineers, buildings today require a collaboration between architect and structural engineer (as well as other specialists such as mechanical/HVAC engineers). See Figure 12.

Conclusion
Perhaps as the de gustibus non est disputandum proverb states, matters of taste can never be definitively settled by recourse to all-encompassing principles. That caveat notwithstanding, it is still true that disputing or discussing the elements that can make structures beautiful is not only an enjoyable pastime, it can, as Hugh Blair suggested 200 years ago, develop one’s taste and appreciation for the finer points of the subject.

References Cited


Vitruvius (Marcus Vitruvius Pollio) (c. 15 BC). de Architectura, or Ten Books on Architecture, translated and published in many editions.

Figure 12
The Burj Khalifa, Dubai, United Arab Emirates, architecture and engineering design by SOM. The tapering form, as in the famous tower in Paris designed by Gustave Eiffel, is structurally efficient in resisting wind loads.

All photos by the author unless otherwise noted.
Is Structural Honesty Always Necessary?

It is true that the nonstructural plates on the horizontal struts of the Golden Gate Bridge obscure the structure behind them, rather than expressing that structure. It is also true that the faceted plates are part of a coordinated aesthetic using Art Deco motifs. Readers can ponder whether they prefer the as-built design or a design that would have exposed the trussed structure.
When Should Appearance Trump Reality?

There are many cases where the architecture requires the structure to be somewhat hidden or unexpressed for practical reasons (e.g., providing the enclosure or skin of a building, fireproofing of steel, matching materials or styles of historic architecture of a building or district). There are also cases where the decorative elements not only cover up the actual structure, but intentionally create another impression in the viewer’s mind of how the structure works.
Buildings Come in Various Shapes

Modern structural engineering has made many unusual forms feasible. However, such structural tours de force often come with extra cost. The usable floor area of multi-story buildings typically is decreased when the building does not go straight up with each floor plate stacked on the same shaped one of the story below.
Bridge Towers Of Various Forms, But Fulfilling The Same Structural Function

An X-braced tower configuration is an efficient way to structure a suspension bridge tower. However, to provide clearance for cars and trucks to drive between the legs of the towers, the X-truss system ends up with a discontinuity.
Yumebutai, Awaji Island, Japan; Tadao Ando, architect. The construction is pure structure, or structural sculpture, with no function typical of a building, such as enclosing occupiable space.

Hôtel de la Reine, Paris, one of Catherine de Medici’s building projects. We think of columns as structural elements holding up weight, but this one only holds up thin air.

Can Structure Be Useless?

Does structure have to serve a purpose, and serve it efficiently? Or can a structure be in and of itself an aesthetic plus, without having any utilitarian reason for existence?
The Art of Structures

Some structures have been considered artful, as well as efficient, expressing how they channel forces to enable them to stand up while also being admired for their aesthetic qualities.
Historic Architecture With Clearly Expressed Structure

In previous centuries, prior to when structural engineering knowledge and modern materials such as steel and reinforced concrete were available, structural solutions often tended to be direct rather than complex. In modern construction, steel can be hidden within the walls, columns, and horizontal spanning elements, but in historic masonry structures, often the visible masonry is the sole means of support.

Historic examples of this include the Santa Sofia/Ayasofya/Hagia Sophia in Istanbul, Turkey, where outward thrust is generated by the dome, whose span was not exceeded until the 1800s. Half domes counteract the thrust at two ends while a large pair of buttresses on each side brace the other direction.

The Wells Cathedral in Wells, United Kingdom, also showcases an unusual use of stone to form inclined columns or struts. The inverted arch was added not for visual effect but to provide bracing under the heavy tower when foundation settlement began to occur.
The Structural Art of Arches

In these examples, the pin-jointed condition at the bases is revealed, and the arches taper down to those points where there is no moment. The articulation of the structural elements and their roles is often cited as a criterion to judge the elegance of a bridge.

Dolan Creek Bridge in northern California, a timber truss bridge with articulated structure, is all based on efficiency. The arch is three-pinned (easier to analyze, more tolerant of foundation settlement), with visible pin joints, as shown in this detailed wood model. The deck over the arch is supported at close intervals; on the approach spans to the arch, a deep lattice truss was needed -- introducing a visual discontinuity, although one that has a structural rationale. The model was made by T. K. May for the 1939 World's Fair.

Shown are two bridges designed by Gustave Eiffel: Pia Maria Bridge, 1877 [on top]; and the Garabit Viaduct, 1884. While very similar in layout, the Garabit Viaduct has an arch that is framed under the deck. In the Pia Maria Bridge, the arch and deck framing merge. Which is more beautiful or makes the structure read more clearly?
What's this? It's not the typing that results when your cat walks across your computer keyboard, it is the anagram by Robert Hooke (1635-1703) that coded his discovery of what today is called funicular analysis of structures, and when the letters are re-arranged properly, it spells out in Latin *ut pendet continuum flexile sic stabit contiguum rigidum inversum*, which means that a flexible chain or cable drooped into a U shape, and loaded with weights representing any other loads that will be on the structure, will, if inverted, form the shape of an arch that will be in pure compression, without any bending.

La Padrera, Barcelona, Antonio Gaudi. If the self-weight of the roof (see the inserted chain image in the inverted image) is the only load, that uniform load along the length of the chain forms a catenary. If (as in a suspension bridge), the cable weight is small while a horizontal deck with traffic loads is large, the load is uniformly distributed along the span of the cable, not along the curving length of the cable, forming a parabola. Arches based on the funicular principle are somewhat pointed (though this is hard to detect if the rise is small compared to the span) and are more efficient than those that are formed from a segment of a circle.

This experimental method was used to determine the form of the roof of the La Sagrada Familia cathedral in Barcelona, designed by Antonio Gaudi (1852-1926). His large model with hanging weights is still on view in the basement of the building. Where arches exerted lateral thrust, columns were tilted to resist the horizontal and vertical forces head on.

© Bernard Gagnon / Wikimedia Commons / CC-BY-SA-3.0
The Beauty and Function of Columns

Must all columns be practical, sensible, and efficient for the structural loads and the architectural floor plans?

Must columns always be sensible structural elements, or can they sometimes be whimsical?

Columns in the middle of doorways and rooms would seem to be inconvenient, but can they also add architectural interest and draw attention to themselves as beautiful sculpture?
Seismic retrofit, University of Tokyo: would you prefer an office on the fourth floor, where no diagonal braces were needed?

Both retail buildings (Tokyo at top, Seattle at bottom) have prominent braced frames in their storefront windows. Which do you prefer? The polished metal finish or the "iron works" look?

**Diagonal Braces Versus Architectural Layouts**

Braced frames can be efficient seismic force-resisting elements, but their diagonal braces sometimes conflict with architectural layouts, especially windows.
Shouldn't These Columns Buckle?

In 1757, Leonard Euler produced this formula to predict the buckling load of a column. (The k end condition factor is not included here).

\[ P = \frac{\pi^2 E I}{L^2} \]

For a given material stiffness (\(E\), modulus of elasticity) and cross-sectional geometry (\(I\), moment of inertia), the buckling load reduces as the square of the length: very slender columns have very low buckling loads.