Bayesian Fragility for Nonstructural Systems

By Chang Hoon Lee and Mircea D. Grigoriu

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Simulation of the Seismic Performance of
Nonstructural Systems

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Chang Hoon Lee¹ and Mircea Grigoriu²

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Project Overview

NEES Nonstructural: Simulation of the Seismic Performance of Nonstructural Systems

Nonstructural systems represent 75% of the loss exposure of U.S. buildings to earthquakes, and account for over 78% of the total estimated national annualized earthquake loss. A very widely used nonstructural system, which represents a significant investment, is the ceiling-piping-partition system. Past earthquakes and numerical modeling considering potential earthquake scenarios show that the damage to this system and other nonstructural components causes the preponderance of U.S. earthquake losses. Nevertheless, due to the lack of system-level research studies, its seismic response is poorly understood. Consequently, its seismic performance contributes to increased failure probabilities and damage consequences, loss of function, and potential for injuries. All these factors contribute to decreased seismic resilience of both individual buildings and entire communities.

Ceiling-piping-partition systems consist of several components, such as connections of partitions to the structure, and subsystems, namely the ceiling, piping, and partition systems. These systems have complex three-dimensional geometries and complicated boundary conditions because of their multiple attachment points to the main structure, and are spread over large areas in all directions. Their seismic response, their interaction with the structural system they are suspended from or attached to, and their failure mechanisms are not well understood. Moreover, their damage levels and fragilities are poorly defined due to the lack of system-level experimental studies and modeling capability. Their seismic behavior cannot be dependably analyzed and predicted due to a lack of numerical simulation tools. In addition, modern protective technologies, which are readily used in structural systems, are typically not applied to these systems.

This project sought to integrate multidisciplinary system-level studies to develop, for the first time, a simulation capability and implementation process to enhance the seismic performance of the ceiling-piping-partition nonstructural system. A comprehensive experimental program using both the University of Nevada, Reno (UNR) and University at Buffalo (UB) NEES Equipment Sites was developed to carry out subsystem and system-level full-scale experiments. The E-Defense facility in Japan was used to carry out a payload project in coordination with Japanese researchers. Integrated with this experimental effort was a numerical simulation program that developed experimentally verified analytical models, established system and subsystem fragility functions, and created visualization tools to provide engineering educators and practitioners with sketch-based modeling capabilities. Public policy investigations were designed to support implementation of the research results.

The systems engineering research carried out in this project will help to move the field to a new level of experimentally validated computer simulation of nonstructural systems and establish a model methodology for future systems engineering studies. A system-level multi-site experimental research plan has resulted in a large-scale tunable test-bed with adjustable dynamic properties, which is useful for future experiments. Subsystem and system level experimental results have produced unique fragility data useful for practitioners.

This report presents a method to calculate the fragility of a nonstructural system supported by a structure subjected to earthquakes. A Bayesian framework is developed for this purpose for a system of gypsum walls.
supported by an 8-story building. The analysis accounts for the uncertainty in the behavior of gypsum walls, which is modeled to match the experimental results. Fragilities are plotted against scale factors corresponding to PGA, PSa, and PSd. ATC-58 fragilities are also calculated and plotted. The proposed method is applicable to other nonstructural system such as piping and ceiling, provided that the associated definitions of damage states and the corresponding experimental data sets are available.

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ABSTRACT

A method is developed for calculating the fragility of a nonstructural system supported by a structure subjected to earthquakes. The nonstructural system consists of a collection of nominally identical components. The input to these components depends on properties of the supporting structure and of site seismicity. It is assumed that (1) the seismic load can be described by 22 ground acceleration records and (2) the components of nonstructural systems have uncertain properties. A Bayesian framework is developed for fragility analysis.

The document provides a manual clarifying the use of the MATLAB code for the developed Bayesian fragility computation. A 2-story perimeter concrete steel moment resisting frame (SMRF) is used to illustrate the use of the code.
ACKNOWLEDGEMENT

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## CONTENTS

<table>
<thead>
<tr>
<th>SECTION</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>GYPSUM WALL FRAGILITY</td>
<td>3</td>
</tr>
<tr>
<td>2.1</td>
<td>Data Set</td>
<td>3</td>
</tr>
<tr>
<td>2.2</td>
<td>Probabilistic Model for ((D_1, D_2, D_3))</td>
<td>5</td>
</tr>
<tr>
<td>2.3</td>
<td>Prior-Posterior Analysis</td>
<td>7</td>
</tr>
<tr>
<td>2.4</td>
<td>Fragility</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>GYPSUM WALL SYSTEM FRAGILITY</td>
<td>13</td>
</tr>
<tr>
<td>3.1</td>
<td>Drift Demand</td>
<td>13</td>
</tr>
<tr>
<td>3.2</td>
<td>Fragility</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>CONCLUSION</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>REFERENCES</td>
<td>21</td>
</tr>
<tr>
<td>A</td>
<td>USERS' MANUAL</td>
<td>23</td>
</tr>
<tr>
<td>A.1</td>
<td>Input</td>
<td>23</td>
</tr>
<tr>
<td>A.1.1</td>
<td>Experimental Data</td>
<td>23</td>
</tr>
<tr>
<td>A.1.2</td>
<td>Simulation Data</td>
<td>25</td>
</tr>
<tr>
<td>A.1.3</td>
<td>Parameters of Probabilistic Model for Damage States</td>
<td>26</td>
</tr>
<tr>
<td>A.1.4</td>
<td>Data of Ground Motions</td>
<td>27</td>
</tr>
<tr>
<td>A.2</td>
<td>Procedure to calculate probabilistic density function and fragility</td>
<td>28</td>
</tr>
<tr>
<td>A.3</td>
<td>Output</td>
<td>30</td>
</tr>
<tr>
<td>A.4</td>
<td>Comments</td>
<td>33</td>
</tr>
<tr>
<td>A.5</td>
<td>List of MATLAB Subroutine Files</td>
<td>34</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Measurements and range of damage states for individual specimens</td>
<td>4</td>
</tr>
<tr>
<td>2.2</td>
<td>Quasi-static fragility testing protocol</td>
<td>4</td>
</tr>
<tr>
<td>2.3</td>
<td>Samples of ((D_1, D_2)) and ((D_2, D_3))</td>
<td>6</td>
</tr>
<tr>
<td>2.4</td>
<td>Marginal predictive densities, (\hat{f}(d_1, d_2)) and (\hat{f}(d_2, d_3)), for state-independent shape parameters</td>
<td>9</td>
</tr>
<tr>
<td>2.5</td>
<td>Marginal predictive densities, (\hat{f}(d_1, d_2)) and (\hat{f}(d_2, d_3)), for state-dependent shape parameters</td>
<td>10</td>
</tr>
<tr>
<td>2.6</td>
<td>Wall fragilities for gypsum walls given by Eq.(2.12) for probabilistic models with state-independent (left panel) and state-dependent (right panel) shape parameters</td>
<td>11</td>
</tr>
<tr>
<td>3.1</td>
<td>Gypsum wall system fragilities for state-independent (left panels) and state-dependent (right panels) shape parameters obtained from Eq.(3.1)</td>
<td>15</td>
</tr>
<tr>
<td>3.2</td>
<td>Gypsum wall system fragilities for state-independent (left panels) and state-dependent (right panels) shape parameters obtained from Eq.(3.2)</td>
<td>16</td>
</tr>
<tr>
<td>3.3</td>
<td>Comparison of ATC-58 with Bayesian fragilities by the model with state-independent shape parameters for the gypsum wall system supported by an 8-story concrete building</td>
<td>17</td>
</tr>
<tr>
<td>A.1</td>
<td>Algorithm for Bayesian fragility analysis and the associated subroutines, input, and output variables</td>
<td>24</td>
</tr>
<tr>
<td>A.2</td>
<td>Fragility for a two-story building by predictive density function with state-independent shape parameters</td>
<td>32</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Definitions of damage states (Filiatrault et al., 2010)</td>
<td>3</td>
</tr>
<tr>
<td>2.2</td>
<td>Damage states measured by University at Buffalo</td>
<td>5</td>
</tr>
</tbody>
</table>
SECTION 1
INTRODUCTION

Fragilities are conditional probabilities that a structural or nonstructural system enters a specified state under a seismic action of specified intensity. Alternative measures have been proposed for seismic intensity, for example, peak ground acceleration and spectral acceleration ordinates. Simplicity is the major advantage of these measures for seismic intensity. However, they may be unsatisfactory when dealing with nonlinear systems (Kafali and Grigoriu, 2007). It has been proposed in Kafali and Grigoriu (2007) to plot fragilities against two parameters, site-to-source distance and earthquake moment magnitude. The resulting plots are surfaces rather than curves, and are referred to as fragility surfaces. If the model in Papageorgiou and Aki (1983) is used to describe the seismic environment at a site, site-to-source distance and earthquake moment magnitude characterize completely the probability law of the seismic ground acceleration process. Fragility surfaces with respect to two spectral ordinates have also been proposed in Baker and Cornell (2005).

Developments in Gardoni et al. (2002) constitute a new direction in fragility analysis. The main idea is to augment current mechanics-based, deterministic models for capacity such that they can capture differences between the performance of nominally identical specimens. Resulting models are probabilistic, have specified functional forms, and depend on a finite number of unknown parameters. These parameters are viewed as random variables whose probability law is inferred from observations via Bayesian analysis.

Fragilities developed in this report are for nonstructural systems with nominally identical components supported by deterministic structures, and are defined as the fraction of components in a nonstructural system that enter a specified damage state as a function of an earthquake intensity measure. The nonstructural system considered in the analysis consists of gypsum walls in an eight story structure. The construction of fragility for the system of gypsum walls in this structure involves (1) dynamic analyses of the supporting structure subjected to a collection of seismic ground motions providing drift time histories at all gypsum walls, (2) experiments performed at the University at Buffalo on gypsum walls identifying three limit states for the walls, and the observation that limit states for each specimen are ordered in the sense that damage state 1 occurs at a drift smaller than that for damage state 2, and so on, and (3) Bayesian characterization of the damage states that are uncertain.

In principle, the approach in Gardoni et al. (2002) can be followed to construct probabilistic models characterizing the behavior of gypsum walls. However, the approach has not been followed since (1) there is no simple deterministic model relating, for example,
imposed drift to damage state for gypsum walls and (2) the main purpose herein is to calculate the probability that gypsum walls enter various damage states under seismic loads.

The associated algorithm is written by MATLAB functions, and its instruction is described with an example of 2-story SMRF in Appendix A.
SECTION 2
GYPSUM WALL FRAGILITY

Consider a population of nominally identical gypsum walls, that is, gypsum walls built in the same manner with the same materials and constrained by the same boundary conditions. Distinct walls of this population are likely to perform differently; for example, some walls may be undamaged and others may enter various damage states under an imposed drift $d$.

Let $D_1, \ldots, D_r$ be values of demand $d$ at which a gypsum wall selected at random enters damage state $r = 0, 1, \ldots, r$. The levels $\{D_r\}$ vary from wall to wall so that they can be viewed as random variables. This research considers $r = 3$ damage states with $r = 0$ denoting the undamaged and $r = 3$ corresponding to damage beyond repair. The damage states are defined on the basis of descriptive failure, as summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Damage state</th>
<th>Description of damage associated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>Superficial damage to the walls</td>
</tr>
<tr>
<td></td>
<td>- Cracks along cornerboards</td>
</tr>
<tr>
<td></td>
<td>- Cracks along joint paper tape</td>
</tr>
<tr>
<td></td>
<td>- Screws pulled out from connections of gypsum boards to steel framing</td>
</tr>
<tr>
<td>$D_2$</td>
<td>Local damage of gypsum wallboards and/or steel frame components</td>
</tr>
<tr>
<td></td>
<td>- Crushing of wall corners</td>
</tr>
<tr>
<td></td>
<td>- Out-of-plane bending and cracking of gypsum wallboards at wall intersections</td>
</tr>
<tr>
<td></td>
<td>- Damage of screws connecting wallboards to boundary studs</td>
</tr>
<tr>
<td></td>
<td>- Bending of boundary studs</td>
</tr>
<tr>
<td></td>
<td>- Buckling of diagonal braces</td>
</tr>
<tr>
<td></td>
<td>- Damage of gypsum wallboards around ceiling connectors or damage induced by ceiling impact</td>
</tr>
<tr>
<td>$D_3$</td>
<td>Severe damage to walls</td>
</tr>
<tr>
<td></td>
<td>- Tears in steel tracks around connectors of track to concrete slab</td>
</tr>
<tr>
<td></td>
<td>- Track fasteners passing through track webs</td>
</tr>
<tr>
<td></td>
<td>- Track flanges bent at wall intersections</td>
</tr>
<tr>
<td></td>
<td>- Hinges forming in studs</td>
</tr>
<tr>
<td></td>
<td>- Partition wall collapse</td>
</tr>
</tbody>
</table>

2.1 Data Set

The left panel of Fig 2.1 shows the range of interstory drifts corresponding to all damage states for all specimens. The right panel of this figure gives the range of damage states
for all tested specimens. Note that the interstory drifts corresponding to distinct damage states overlap. The plots summarize measurements reported in Table 2.2. The red circles in Fig 2.2 indicate instances at which specimens have been inspected (Filiatrault et al., 2010; Retamales et al., 2008).

![Figure 2.1: Measurements and range of damage states for individual specimens](image1)

![Figure 2.2: Quasi-static fragility testing protocol](image2)

Data in Fig. 2.3 and Table 2.2 show that critical drift level \((D_1, D_2, D_3)\) are ordered; that is, they satisfy the condition \(D_1 \leq D_2 \leq D_3\) for each specimen. Note that a gypsum wall selected at random and subjected to a drift level \(d\) enters damage state 0, 1, 2, and 3 with probability \(P(d < D_1)\), \(P(D_1 \leq d < D_2)\), \(P(D_2 \leq d < D_3)\), and \(P(D_3 \leq d)\), respectively.
### Table 2.2: Damage states measured by University at Buffalo

<table>
<thead>
<tr>
<th>Specimen NO</th>
<th>Original</th>
<th>Modified</th>
<th>Scaled</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_1$ (%)</td>
<td>$D_2$ (%)</td>
<td>$D_3$ (%)</td>
</tr>
<tr>
<td>1</td>
<td>0.20</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>0.62</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>0.40</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>4</td>
<td>0.40</td>
<td>0.62</td>
<td>1.16</td>
</tr>
<tr>
<td>5</td>
<td>0.20</td>
<td>0.40</td>
<td>2.32</td>
</tr>
<tr>
<td>6</td>
<td>0.40</td>
<td>0.62</td>
<td>2.66</td>
</tr>
<tr>
<td>7</td>
<td>0.20</td>
<td>0.62</td>
<td>1.00</td>
</tr>
<tr>
<td>8</td>
<td>0.40</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>9</td>
<td>0.20</td>
<td>0.40</td>
<td>0.62</td>
</tr>
<tr>
<td>10</td>
<td>0.20</td>
<td>0.81</td>
<td>0.81</td>
</tr>
<tr>
<td>17</td>
<td>0.81</td>
<td>0.81</td>
<td>1.99</td>
</tr>
<tr>
<td>18</td>
<td>0.81</td>
<td>0.81</td>
<td>1.84</td>
</tr>
<tr>
<td>19</td>
<td>0.62</td>
<td>0.81</td>
<td>0.81</td>
</tr>
<tr>
<td>20</td>
<td>0.20</td>
<td>1.00</td>
<td>2.32</td>
</tr>
<tr>
<td>21</td>
<td>0.40</td>
<td>0.81</td>
<td>-</td>
</tr>
<tr>
<td>22</td>
<td>0.62</td>
<td>0.62</td>
<td>1.00</td>
</tr>
<tr>
<td>23</td>
<td>0.40</td>
<td>0.81</td>
<td>1.00</td>
</tr>
<tr>
<td>24</td>
<td>0.40</td>
<td>0.40</td>
<td>1.16</td>
</tr>
<tr>
<td>25</td>
<td>0.40</td>
<td>0.40</td>
<td>0.62</td>
</tr>
<tr>
<td>26</td>
<td>0.40</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>27</td>
<td>0.40</td>
<td>0.62</td>
<td>0.81</td>
</tr>
<tr>
<td>28</td>
<td>0.40</td>
<td>0.81</td>
<td>0.81</td>
</tr>
<tr>
<td>31</td>
<td>0.20</td>
<td>0.62</td>
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</tr>
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<td>32</td>
<td>0.40</td>
<td>1.00</td>
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<td>0.20</td>
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<td>-</td>
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<td>1.00</td>
<td>1.35</td>
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</tr>
<tr>
<td>35</td>
<td>0.20</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>36</td>
<td>0.40</td>
<td>0.62</td>
<td>0.62</td>
</tr>
</tbody>
</table>

**Note:**
1. Specimens 21, 33, and 35 have not been used in analysis because of missing data.
2. The symbol (†) indicates that $D_{i+1}$ is changed to $D_{i+1} = D_i + \epsilon'$, $\epsilon' > 0$, if $D_i = D_{i+1}$.

### 2.2 Probabilistic Model for $(D_1, D_2, D_3)$

Independent samples of critical drift levels $(D_1, D_2, D_3)$ are shown in Fig. 2.3. Critical levels for gypsum walls are bounded and satisfy the condition $D_1 \leq D_2 \leq D_3$, so that they take values in the wedge $W = \{(z_1, z_2, z_3) \in \mathbb{R}^3 : z_1 \leq z_2 \leq z_3\}$. The joint density
Figure 2.3: Samples of \((D_1, D_2)\) and \((D_2, D_3)\)

\[ f(d_1, d_2, d_3) \text{ of } (D_1, D_2, D_3) \text{ can be given in the form } \]

\[ f(d_1, d_2, d_3) = f(d_3 \mid d_2, d_1) \, f(d_2 \mid d_1) \, f(d_1), \quad (2.1) \]

where \(f(\cdot \mid d_2, d_1)\), \(f(\cdot \mid d_1)\), and \(f(\cdot)\) denote the densities of \(D_3 \mid (D_2 = d_2, D_1 = d_1)\), \(D_2 \mid (D_1 = d_1)\), and \(D_1\), respectively. The calculations presented herein used the simplified model

\[ f(d_1, d_2, d_3) \simeq f(d_3 \mid d_2) \, f(d_2 \mid d_1) \, f(d_1), \quad (2.2) \]

obtained by the approximation \(f(d_3 \mid d_2, d_1) \simeq f(d_3 \mid d_2)\).

The subset of \(W\) giving the support of the density of \((D_1, D_2, D_3)\) can be constructed as follows. Let \(d_{r,l}\) and \(d_{r,h}\) be the smallest and the largest recorded values of \(D_r, r = 1, 2, 3\). Since the random variables \(\{D_r\}\) take values in bounded intervals, it is assumed that they can be described by beta distributions with known range but unknown shape parameters. Accordingly, \(D_1\) is assumed to be a beta distribution with range \([\tilde{d}_{1,l}, \tilde{d}_{1,h}] = [d_{1,l} - \varepsilon, d_{1,h} + \varepsilon]\) and unknown shape parameters \((p_1, q_1)\). The range \([d_{1,l}, d_{1,h}]\) is widened to \([\tilde{d}_{1,l}, \tilde{d}_{1,h}] = [d_{1,l} - \varepsilon, d_{1,h} + \varepsilon], \varepsilon > 0\) (\(\varepsilon = 0.005\%\) is used in the analysis), to allow for values of \(D_1\) outside currently available observations. A similar approach is used for the other critical drift levels. The conditional random variable \(D_2 \mid (D_1 = d_1)\) is assumed to follow a beta distribution with range \([\tilde{d}_{2,l}, \tilde{d}_{2,h}] = [\max(d_{2,l}, d_{1,l}) - \varepsilon, d_{2,h} + \varepsilon]\) and shape parameters \((p_2, q_2)\) that may depend on \(d_1\). The conditional random variable \(D_3 \mid (D_2 = d_2)\) is assumed to be a beta distribution with range \([\tilde{d}_{3,l}, \tilde{d}_{3,h}] = [\max(d_{3,l}, d_{2,l}) - \varepsilon, d_{3,h} + \varepsilon]\) and shape parameters \((p_3, q_3)\) that may depend on \(d_2\).
It is convenient to work with the random variables
\[
D_r = \frac{D_r - \bar{d}_{r,l}}{d_{r,h} - d_{r,l}} \in [0, 1], \quad r = 1, 2, 3,
\] (2.3)
that are standard beta distributions since, if \(X\) is a beta random variable with range \([a, b]\) and shape parameters \((p, q)\), the random variable \(Y = (X - a)/(b - a)\) is a standard beta variables with shape parameters \((p, q)\) and density
\[
f_Y(y) = \frac{y^{p-1} (1 - y)^{q-1}}{B(p, q)}, \quad 0 \leq y \leq 1,
\] (2.4)
where \(B(p, q)\) denotes the beta function and \(p, q > 0\). Observations in Fig. 2.3 can be scaled by Eq. 2.3 and used to infer properties of random variables \(\{\bar{D}_r\}\). Table 2.2 lists both the observations in Fig. 2.3 and their scaled version \((e_{1,s}, e_{2,s}, e_{3,s}), s = 1, \ldots, n_s\), given by Eq. 2.3.

2.3 Prior-Posterior Analysis

Since no information is available on the shape parameters of the beta distributions postulated for random variables \(\{D_r\}\), it is assumed that these parameters are uniformly distributed in some bounded sets that depend on whether the potential dependence of the shape parameters \((p_r, q_r)\) on \(D_{r-1}, r = 2, 3\), is or is not modeled.

If the shape parameters \((p_r, q_r)\) for \(D_2 \mid D_1\) and \(D_3 \mid D_2\) are assumed to be independent of \(D_1\) and \(D_2\), respectively, the prior density of the uncertain parameters \(\theta = (p_1, q_1, p_2, q_2, p_3, q_3)\) in the definition of the joint density of \((D_1, D_2, D_3)\) is
\[
f'(\theta) = \prod_{r=1}^{3} \frac{1}{(p_{r,h} - p_{r,l})(q_{r,h} - q_{r,l})}, \quad (p_r, q_r) \in [p_{r,l}, p_{r,h}] \times [q_{r,l}, q_{r,h}],
\] (2.5)
where \(0 < p_{r,l} < p_{r,h}\) and \(0 < q_{r,l} < q_{r,h}\), so that the posterior density of \(\theta\) has the form
\[
f''(\theta) \propto f'(\theta) \ell(\theta \mid \text{data}),
\] (2.6)
where
\[
\ell(\theta \mid \text{data}) = \prod_{s=1}^{n_s} f(e_{3,s} \mid e_{3,s}) f(e_{2,s} \mid e_{1,s}) f(e_{1,s})
\] (2.7)
denotes the likelihood function using the scaled observations \(\{e_{r,s}\}, s = 1, \ldots, n_s\), where we use the same notation as in Eq. 2.1 although scaled rather than actual measurements.
are used.

The potential dependence of \((p_r, q_r)\) on \(D_{r-1}\), \(r = 2, 3\), can be modeled simply by partitioning the range of \(D_{r-1}\) in two or more intervals and postulating that \((p_r, q_r)\) are constant in these intervals. For example, suppose the ranges of \(D_1\) and \(D_2\) are partitioned in the intervals \((I_{1,1}, I_{1,2})\) and \((I_{2,1}, I_{2,2})\) and assume that shape parameters are constant in these intervals; that is, they are \((p_{2,1}, q_{2,1})\) and \((p_{2,2}, q_{2,2})\) in \(I_{1,1}\) and \(I_{1,2}\), and \((p_{3,1}, q_{3,1})\) and \((p_{3,2}, q_{3,2})\) in \(I_{2,1}\) and \(I_{2,2}\), respectively. In this case, the vector of uncertain parameters is \(\theta = (p_1, q_1, p_{2,1}, q_{2,1}, p_{2,2}, q_{2,2}, p_{3,1}, q_{3,1}, p_{3,2}, q_{3,2})\). The prior density on \(\theta\) can be as in Eq. 2.5, that is, it is assumed that the coordinates of \(\theta\) are independent, \(p_{r,1}, p_{r,2}\) are uniformly distributed in \([p_{r,1}, p_{r,2}]\), and \(q_{r,1}, q_{r,2}\) are uniformly distributed in \([q_{r,1}, q_{r,2}]\) for \(r = 2, 3\). The posterior density \(f''(\theta)\) is given by Eq. 2.6 with

\[
\ell(\theta \mid \text{data}) = \prod_{s=1}^{n_s} f(e_{3,s} \mid e_{2,s}) f(e_{2,s} \mid e_{1,s}) f(e_{1,s}) \tag{2.8}
\]

where \(f(e_{1,s}) = e_{1,s}^{p_{1,s} - 1} (1 - e_{1,s})^{q_{1,s} - 1} / B(p_1, q_1)\), \(f(e_{2,s} \mid e_{1,s}) = e_{2,s}^{p_{2,s} - 1} (1 - e_{2,s})^{q_{2,s} - 1} / B(p_u, q_u)\) with \(u = 1\) if \(e_{1,s} \in I_{1,1}\) and \(u = 2\) if \(e_{1,s} \in I_{1,2}\), and \(f(e_{3,s} \mid e_{2,s}) = e_{3,s}^{p_{3,s} - 1} (1 - e_{3,s})^{q_{3,s} - 1} / B(p_u, q_u)\) with \(u = 1\) if \(e_{2,s} \in I_{2,1}\) and \(u = 2\) if \(e_{2,s} \in I_{2,2}\).

The predictive density \(\hat{f}(d_1, d_2, d_3)\) can be obtained from the density of \((D_1, D_2, D_3)\) in Eq. 2.2 by eliminating the condition on the shape parameters \(\theta\) in the expression of \(f(d_1, d_2, d_2)\), that is,

\[
\hat{f}(d_1, d_2, d_3) = \int f(d_1, d_2, d_2) f''(\theta) d\theta. \tag{2.9}
\]

The integral in this equation has a dimension of 6 if \((p_r, q_r)\) are assumed to be invariant with \(D_{r-1}\), \(r = 2, 3\), and a dimension of 10 if \((p_r, q_r)\) are allowed to take two distinct values in the range of \(D_{r-1}\), \(r = 2, 3\). To simplify calculations, the integration in Eq. 2.9 is performed by Monte Carlo simulation (Liu, 2001).

Fig. 2.4 and 2.5 show marginal predictive densities, \(\hat{f}(d_1, d_2)\) and \(\hat{f}(d_2, d_3)\), of \(\hat{f}(d_1, d_2, d_3)\) with state-dependent and -independent shape parameters on \(D_1\) or \(D_2\), respectively, and they were computed from

\[
\hat{f}(d_1, d_2) = \int \hat{f}(d_1, d_2, d_3) dd_3 \tag{2.10}
\]

\[
\hat{f}(d_2, d_3) = \int \hat{f}(d_1, d_2, d_3) dd_1 \tag{2.11}
\]

for both models. The range of shape parameters used for calculations is \((p_r, q_r) \in [0.1, 4] \times [0.1, 4]\). This range has been selected such that it contains the maximum likelihood esti-
Figure 2.4: Marginal predictive densities, \( \hat{f}(d_1, d_2) \) and \( \hat{f}(d_2, d_3) \), for state-independent shape parameters.

mates of \((p, q)\).

2.4 Fragility

Previous sections have defined the probabilities that a gypsum wall enters a damage state \( r \) if subjected to drift \( d \). The posterior densities \( f''(\theta) \) for the uncertain parameters \( \theta \) in the definition of the probability law of \((D_1, D_2, D_3)\) has been used to develop the predictive density \( \hat{f}(d_1, d_2, d_3) \) for \((D_1, D_2, D_3)\).

Gypsum wall fragilities are plots of the probabilities of entering damage state \( r \) as a function of drift demand \( d \). For example, fragility for damage state \( r \) is

\[
p_r(d) = \int P(D_r \leq d < D_{r+1}) f''(\theta) \, d\theta, \quad r = 0, 1, 2, 3
\] (2.12)
Figure 2.5: Marginal predictive densities, $\hat{f}(d_1, d_2)$ and $\hat{f}(d_2, d_3)$, for state-dependent shape parameters

with the convention $P(D_3 \leq d < D_4) = P(D_3 \leq d)$.

These fragilities are shown in Fig 2.6 for state-independent (left panel) and state-dependent (right panel) shape parameters. Note that $p_0(d)$ is an increasing function of $d$. Fragility $p_1(d)$ increases then decreases with $d$, since, for larger $d$, damage is likely to be in damage state 2 or 3.
Figure 2.6: Wall fragilities for gypsum walls given by Eq.(2.12) for probabilistic models with state-independent (left panel) and state-dependent (right panel) shape parameters
SECTION 3
GYPSUM WALL SYSTEM FRAGILITY

Suppose a system of $n$ gypsum walls is supported by an eight-story building subjected to seismic loads. It is assumed that the structure is deterministic with known properties, site seismicity can be described by a collection $a_i(t)$, $i = 1, \ldots, m$, of ground accelerations, and cascade analysis holds; that is, these gypsum walls do not affect structural behavior. The purpose herein is to assess the seismic performance of the gypsum wall system.

3.1 Drift Demand

Let $\tilde{a}_i(t) = a_i(t)/\text{PGA}_i$, $i = 1, \ldots, m$, denote scaled ground accelerations, where $\text{PGA}_i = \max_t |a_i(t)|$ is the peak ground acceleration of $a_i(t)$. Seismic intensity measures superior to peak ground acceleration are available (Baker and Cornell, 2005; Kafali and Grigoriu, 2007). Spectral acceleration and displacements are also used to characterize seismic intensity, in addition to the peak ground acceleration in fragility calculation.

Denote by $d_{i,k}(\xi)$ the drift demand on wall $k = 1, \ldots, n$ of a gypsum wall system supported by a deterministic structure subjected to a collection of ground motions $\tilde{a}_i(t)$, $i = 1, \ldots, m$, where $\xi > 0$ indicates a scale factor. The scale factor $\xi$ can be related to PGA or spectral values.

3.2 Fragility

Fragilities of individual gypsum walls can be used to calculate fragilities for systems of such walls, for example, a system of $n$ nominally identical gypsum walls supported by a deterministic structure. System fragility depends on the particular definition used for system damage.

As for individual gypsum walls, four damage states for gypsum wall systems are considered. The definition of system damage states is not unique and depends on the objective of the analysis. The following definition is used in this discussion. A gypsum wall system is in damage state 0 if all its components are undamaged; it enters damage state 1, 2, and 3 if at least one of its walls is in damage state 1 or higher, 2 or higher, and 3.

Two limit cases are considered. First, suppose wall properties are independent and follow the same distribution; that is, critical drift levels $(D_{1,k}, D_{2,k}, D_{3,k})$ for walls $k = 1, \ldots, n$ are independent copies of $(D_1, D_2, D_3)$. Under this assumption, the system of
gypsum walls enters damage state 0, 1, 2, and 3 with probabilities

\begin{align*}
\text{Damage state 0 : } & p_{0,i}(\xi) = \prod_{k=1}^{n} P(d_{i,k}(\xi) < D_{1,k}), \\
\text{Damage state 1 : } & p_{1,i}(\xi) = 1 - \prod_{k=1}^{n} P(d_{i,k}(\xi) < D_{1,k}), \\
\text{Damage state 2 : } & p_{2,i}(\xi) = 1 - \prod_{k=1}^{n} P(d_{i,k}(\xi) < D_{2,k}), \quad \text{and} \\
\text{Damage state 3 : } & p_{3,i}(\xi) = 1 - \prod_{k=1}^{n} P(d_{i,k}(\xi) < D_{3,k}). \quad (3.1)
\end{align*}

Second, suppose wall properties are perfectly correlated, meaning that all walls have the same critical drift levels represented by random variables \((D_1, D_2, D_3)\). Under this assumption, the system of gypsum walls enters damage state 0, 1, 2, and 3 with probabilities

\begin{align*}
\text{Damage state 0 : } & p_{0,i}(\xi) = P\left( \max_{1 \leq k \leq n} d_{i,k}(\xi) < D_1 \right), \\
\text{Damage state 1 : } & p_{1,i}(\xi) = P\left( \max_{1 \leq k \leq n} d_{i,k}(\xi) \geq D_1 \right), \\
\text{Damage state 2 : } & p_{2,i}(\xi) = P\left( \max_{1 \leq k \leq n} d_{i,k}(\xi) \geq D_2 \right), \quad \text{and} \\
\text{Damage state 3 : } & p_{3,i}(\xi) = P\left( \max_{1 \leq k \leq n} d_{i,k}(\xi) \geq D_3 \right). \quad (3.2)
\end{align*}

Note that the probabilities \(p_{0,i}(\xi), p_{1,i}(\xi), p_{2,i}(\xi),\) and \(p_{3,i}(\xi)\) depend on the probability law of critical drift levels in addition ground motion \(\tilde{a}_i(t)\) and the scale factor on this motion. Since ground motions are assumed to be equally likely, system fragilities can be obtained from the probabilities

\[ p_r(\xi) = \frac{1}{m} \sum_{i=1}^{m} p_{r,i}(\xi), \quad r = 0, 1, 2, 3. \quad (3.3) \]

The probabilities in Eqs. 3.1 and 3.2 can be calculated since the range of random variables \(\{D_r\}\) has been specified and the posterior densities of the shape parameters is given by Eq. 2.6. Resulting probabilities deliver fragilities for systems of gypsum walls. Figure 3.1 shows system fragilities for state-independent (left panel) and state-dependent (right panel) shape parameters calculated from Eq.(3.1). Fragilities given by Eq.(3.2) are plotted in a similar manner in Fig. 3.2.

Figure 3.3 shows ATC-58 and Bayesian fragilities for damage states \(D_0\) (undamaged
Figure 3.1: Gypsum wall system fragilities for state-independent (left panels) and state-dependent (right panels) shape parameters obtained from Eq.(3.1)
Figure 3.2: Gypsum wall system fragilities for state-independent (left panels) and state-dependent (right panels) shape parameters obtained from Eq.(3.2)
system), $D_1$, $D_2$, and $D_3$ for a gypsum wall system supported by an 8-story concrete building.

![Graphs showing fragility curves for ATC-58 and Bayesian models for different D values](image)

**Figure 3.3:** Comparison of ATC-58 with Bayesian fragilities by the model with state-independent shape parameters for the gypsum wall system supported by an 8-story concrete building
SECTION 4
CONCLUSION

A Bayesian framework has been developed for calculating fragilities for a system of gypsum walls supported by an 8-story building subjected to an earthquake. The analysis accounts for the uncertainty in the behavior of gypsum walls, that is modeled to match experimental results. Fragilities have been plotted against scale factors corresponding to $PGA$, $PS_a$, and $PS_d$. ATC-58 fragilities have also been calculated and plotted.

The proposed method is applicable to other nonstructural systems, such as piping and suspended ceilings, provided that the associated definitions of damage states and the corresponding experimental data sets would be given in the future.


A.1 Input

Input variables of MATLAB code include:

- Experimental data of damage states (University at Buffalo),
- Simulation data (University of California at San Diego),
- Parameters of probabilistic model for damage states, and
- Ground motion records.

Input command is:

\[
\text{bayesfragility(data,epsilon,tole,nodiv,ns,nx1,nx2,nx3,...}
\text{extd,drift,intensity,n_grm,td,zeta)}
\]

The algorithm is shown in Figure A.1.

A.1.1 Experimental Data

**Format:** The experimental data of damage states are stored in an \((n_{sp}, n_{damage})\) matrix called “data.txt”, where \(n_{sp}=\) number of specimens and \(n_{damage}=\) number of damage states.

- If \(D_k = D_{k+1}\), set \(D_{k+1} = D_{k+1} + \text{tole}\), where \(\text{tole} > 0\) is an input. This mapping is automatically performed in the code and the result is stored in matrix \(ds\) (line 30 – 51 in the code). Both original and modified data are shown in Table 2.2.

- To run the code, execute the following command.

```
data = load('data.txt'); % n_sp = 25, no_damage = 3
```

**Example:**

```
data =

0.2000  0.6200  0.6200
0.2000  0.6200  1.0000
```
Figure A.1: Algorithm for Bayesian fragility analysis and the associated sub-routines, input, and output variables

\[
\begin{array}{ccc}
0.4000 & 0.6200 & 0.6200 \\
0.4000 & 0.6200 & 1.1600 \\
0.2000 & 0.4000 & 2.3200 \\
0.4000 & 0.6200 & 2.6600 \\
0.2000 & 0.6200 & 1.0000 \\
0.4000 & 1.0000 & 1.0000 \\
0.2000 & 0.4000 & 0.6200 \\
\end{array}
\]

--- shown from row 1 to row 9 of data

\[
ds =
\begin{array}{ccc}
0.2000 & 0.6200 & 0.6300 \\
0.2000 & 0.6200 & 1.0000 \\
0.4000 & 0.6200 & 0.6300 \\
0.4000 & 0.6200 & 1.1600 \\
\end{array}
\]
A.1.2 Simulation Data

Format: The inter-story drift data is stored in a \((n_{grm} \times n_{pga}, n_{story}+2)\) matrix called “drift.txt”, where \(n_{grm}\) = the number of ground motions, \(n_{pga}\) = number of peak ground acceleration, and \(n_{story}\) = number of stories for a building.

- First column: Number of ground motion
- Second column: Peak ground acceleration
- Column 3 to Column \(n_{story} + 2\): Inter-story drifts for the first, second, ..., story.
- Row 1 to row \(n_{grm}\) and row \(n_{grm}+1\) to row \(2 \times n_{grm}\) at column 3 to \(n_{story} + 2\) gives inter-story drifts for the first PGA, and so on.
- To run the code, execute

\[
\text{drift} = \text{load}('drift.txt'); \quad \% n_{grm} = 22, n_{story} = 2
\]

Example:

\[
\text{drift} = \\
\begin{array}{cccc}
1.0000 & 0.0500 & 0.0115 & 0.0170 \\
2.0000 & 0.0500 & 0.0132 & 0.0197 \\
3.0000 & 0.0500 & 0.0067 & 0.0099 \\
4.0000 & 0.0500 & 0.0095 & 0.0140 \\
5.0000 & 0.0500 & 0.0116 & 0.0178 \\
6.0000 & 0.0500 & 0.0184 & 0.0273 \\
7.0000 & 0.0500 & 0.0090 & 0.0129 \\
8.0000 & 0.0500 & 0.0109 & 0.0163 \\
9.0000 & 0.0500 & 0.0092 & 0.0131 \\
\end{array}
\]
A.1.3 Parameters of Probabilistic Model for Damage States

- \( \text{epsilon} \): Extend the range of data set \( ds \) from \( \{\min(ds_k), \max(ds_k)\}, k = 1, 2, 3 \) to
  \[ \{\min(ds_k) - \epsilon, \max(ds_k) + \epsilon\}, k = 1, 2, 3 \] (A.1)

  for computational reasons, e.g., \( \epsilon = 0.005 \) can be taken. This is explained in Section 2.2 Probabilistic Model for \((D_1, D_2, D_3)\) of the report.

- \( \text{nodiv} \): Number of partition for the range of \( ds(:,r-1) \), \( r = 2, 3 \), to allow variable values for \((p_r, q_r)\), e.g., \( \text{nodiv}=2 \) that indicates 2 partitions are divided in \( ds_1 \) and \( ds_2 \) when \( \text{no_damage}=3 \).
  
  - If \( \text{nodiv}=1 \), no division exists in the damage state.

- \( \text{ns} \): Number of Monte-Carlo samples, e.g., \( \text{ns}=10000 \).

- \((nx_1, nx_2, nx_3)\): Number of mesh divisions for the predictive density function of damage states, e.g., \( nx=30, ny=30, nz=30 \) depending on \( \text{no_damage} \).

- \( \text{extd} \): the support of prior density for \((p_r, q_r)\), assumed to be uniform in rectangular domain, e.g., \( \text{extd}=10 \).

- \( \text{n.grm} \): the number of ground motion used in the numerical analysis, e.g., \( \text{n.grm}=22 \) from simulation data provided by UCSD.
• **Intensity**: type of the ground motion intensity for fragility calculation, e.g., intensity=1: PGA-based, intensity=2: $PS_a$-based, intensity=3: $PS_d$-based fragilities.

• **td**: natural period of the given system, e.g., td=0.196 second for RC-2 (2-story SMRF building).

• **zeta**: damping ratio of the system, e.g., zeta=0.05. Although a user selects to calculate the fragility as a function of spectral response, zeta is still required.

### A.1.4 Data of Ground Motions

Ground motion records are only required when the fragility is computed on the basis of spectral response. The ground motion data and the associated file must be prepared as follows.

**Format**: Each record file must be named “record1.txt”, “record2.txt”, and so on, including the ground acceleration as a column vector. The number of ground motion records must be consistent with $n_{grm}$ input parameter indicating the number of ground motion. The size of time step for the given data must be included in the file “motion_dt.txt” as a column vector. The row number of “motion_dt.txt” must correspond to the record number. The size of time step of “record3.txt” must be stored at the third row of “motion_dt.txt” file.

**Example:**

```matlab
>> record10(40:45)

ans =

1.0e-04 *

0.1874
0.1907
0.1850
0.1839
0.1937
0.2145
```


If intensity is selected to be 2 and 3, the code automatically computes the fragilities as a function of $PS_a$ and $PS_d$, respectively. Subroutine “spectrum_generator.m” is used to map PGA into $PS_a$ and $PS_d$.

A.2 Procedure to calculate probabilistic density function and fragility

The denoted line numbers refer MATLAB code of “bayesianfragility.m” and the associated equation is explained in Section 2.

STEP 1: Modification of data set (Line: 33–51 / Table 2.2)

Example:

ds =

0.2000 0.6200 0.6300
0.2000 0.6200 1.0000
0.4000 0.6200 0.6300
0.4000 0.6200 1.1600
0.2000 0.4000 2.3200
0.4000 0.6200 2.6600

----------------------------- shown from row 1 to 6 of ds

STEP 2: Scaling of data set (Line: 71–109 / Equation 2.3)

- dsscale.m is sub-routine to scale damage states.
- The algorithm initially finds the row numbers to divide damage states if nodiv > 1.
STEP 3: Determination of shape parameter ranges (Line: 111–165 / Section 2.3)

- The maximum shape parameter is extended to a certain range by multiplying $\text{extd}$.
- $\text{prange}$ and $\text{qrange}$ include the ranges of $(p_r, q_r)$. Since $p_r > 0$ and $q_r > 0$, the minimum of shape parameters were set to 0.01.

STEP 4: Calculation of prior density function (Line: 167–173 / Equation 2.5)

- Uniform distribution for the prior density is postulated.

STEP 5: Calculation of posterior density function (Line: 175–179 / Equation 2.6 and 2.7)

- $\text{post}_{\text{mc}}$ is a sub-routine to calculate the posterior density.
- The posterior density function is stored at $\text{fpos}$, and its length corresponds to the number of samples for Monte-Carlo integration $\text{ns}$.

STEP 6: Calculation of predictive density function (Line: 181–242 / Equation 2.9, 2.10, and 2.11)

- $\text{pred}_{\text{mc}}$ is a sub-routine to compute the predictive density function $\hat{f}(d_1)$, $\hat{f}(d_1, d_2)$, or $\hat{f}(d_1, d_2, d_3)$ according to $\text{no}_{\text{damage}}=1$, $\text{no}_{\text{damage}}=2$, and $\text{no}_{\text{damage}}=3$, respectively.
- The calculated predictive density is store at $\text{fpred}_{\text{un}}$. When normalizing the predictive density function, the format is changed to a vector ($\text{no}_{\text{damage}}=1$), matrix ($\text{no}_{\text{damage}}=2$), and 3-D matrix ($\text{no}_{\text{damage}}=3$) (line 231–242 in the code). After normalizing, the predictive density is saved as $\text{fx}$.

**Example:**

```matlab
defx(:, :, 1) =

Columns 1 through 6
85.1429 65.9968 57.4673 51.3481 46.3637 42.0644
```
The resulting marginal densities are shown in Figure 2.5 and 2.4.

STEP 7: Fragility calculation (Line: 271–367 / Equation 3.1, 3.2 and 3.3)

- `frag_wall_indep.m` and `frag_wall_dep.m` are subroutines to calculate fragilities of systems with independent and perfectly correlated wall properties (See Eqs.(3.1) and (3.2) in Section 3). These fragilities are saved at matrices `bayA` and `bayB`, respectively. Fragilities can be plotted against PGA, $PS_a$, and $PS_d$.

A.3 Output

**Format:** The output of Bayesian fragility calculation created the matrix `bayA` and `bayB` and each matrix is ($n_{pga}$, no $damage$ + 2) where $n_{pga}$ is the number of peak ground acceleration and no $damage$ + 2 indicates $no$ $damage$ damage states in addition to zero system fragility and intensity measure of ground motion at the first column of `bayA` and `bayB`.

**Example:**

```matlab
>> bayA(40:43,:) ans =

     2.0000    0.6294    0.3706    0.0592    0.0014
     2.0500    0.6145    0.3855    0.0692    0.0017
     2.1000    0.5963    0.4037    0.0762    0.0020
     2.1500    0.5772    0.4228    0.0855    0.0023
```

--------------------------- shown from row 40 to row 43

```matlab
>> bayB(40:43,:) ans =
```

```matlab
            30
```
2.0000  0.6294  0.5556  0.1433  0.0124
2.0500  0.6145  0.5683  0.1578  0.0124
2.1000  0.5963  0.5938  0.1694  0.0146
2.1500  0.5772  0.6165  0.1793  0.0167

Also, Bayesian fragility can be computed on the basis of different ground motion intensity such as $P_{S_a}$ or $P_{S_d}$. The following results are examples of Bayesian fragility as a function of $P_{S_a}$.

>> bayA(40:43,:)
ans =

4.0952  0.5966  0.4034  0.1163  0.0198
4.1933  0.5386  0.4614  0.1316  0.0269
4.2985  0.5571  0.4429  0.0990  0.0170
4.4031  0.5041  0.4959  0.1499  0.0377

>> bayB(40:43,:)
ans =

4.0952  0.5966  0.5711  0.1710  0.0442
4.1933  0.5386  0.6435  0.1997  0.0549
4.2985  0.5571  0.6104  0.1773  0.0421
4.4031  0.5041  0.6404  0.2394  0.0693

Figure A.2 presents Bayesian fragilities of nonstructural system supported by a 2-story SMRF building. The predictive density function with state-independent shape parameters (no_div=1) is applied, and fragilities are calculated according to two different limit states described in Section 3 (left panels by Eq.(3.1) and right panels by Eq.(3.2)) and different seismic intensities ($PGA$-based (top panels), $P_{S_a}$-based (middle panels) and $P_{S_d}$-based fragilities (lower panels)).
Figure A.2: Fragility for a two-story building by predictive density function with state-independent shape parameters
A.4 Comments

In calculation of Bayesian fragility, about 99% of the computational time is taken by the process to calibrate the probabilistic model and the rest of the time is spent on calculating the system fragility on the basis of a previously given example. If a user tends to calculate fragilities of different systems using a predictive density function, it is efficient to perform several fragility calculations for different systems after a single computation of the probabilistic model calibration.

It is recommended to separate the code in two parts: calibration of the model and calculation of fragility, and to write in two different files as follows.

(1) `calibration_model.m` for the calibration of the probabilistic model and

(2) `fragility_calculation.m` for fragility computation according to different seismic intensities.

The following steps are required to perform the separate analysis.

**STEP 1:** Run `calibration_model.m`. The input is exactly the same as the previous section except the excluded input variables, `drift, n_grm, n_story` that related to the simulation data. Outputs of this code are the predictive density function `fx` and the associated damage state space `xs_i`, `i = 1, 2, 3` according to the number of damage states. Command to run the code is as follows:

```matlab
data = load('data.txt');
calibration_model(data,epsilon,tole,nodiv,ns,...
nx1,nx2,nx3,extd);
```

The code creates the predictive density function (`fxdiv` or `fxdiv` according to consideration of division) and the associated domain (`x1, x2, and x3`).

**STEP 2:** Subsequently, run `fragility_calculation.m` with the input variables `drift, no_damage, intensity, n_grm, td, and zeta`.

```matlab
drift = load('drift.txt');
load fxnodiv; % or load fxdiv;
fragility_calculation(fx,drift,no_damage,intensity,...
n_grm,td,zeta)
```
As a result, the code will create \((n_{\text{pga}}, no_{\text{damage}}+2)\) matrix \(\text{bayA}\) and \(\text{bayB}\) for the system fragility.

The STEP 2 can be repeated for various systems on the basis of the predictive density function \(f_x\) on the domain \((\min(x_1), \max(x_1)) \times (\min(x_2), \max(x_2)) \times (\min(x_3), \max(x_3))\) when \(no_{\text{damage}}=3\) for an example.

### A.5 List of MATLAB Subroutine Files

With the functions provided by MATLAB, the following functions are required to perform the fragility calculation by Bayesian framework.

- **fragility\_calculation.m** is an algorithm to compute Bayesian fragility as a function of PGA, \(PS_a\) and \(PS_d\).

- **randiv*D.m** is an algorithm to find row numbers of sorted data set at dividing the range according to 1, 2, and 3 damage states.\((* = 1,2,3)\)

- **dsscale.m** is an algorithm to scale a damage state measurement to \((0, 1)\) for standard Beta density function according to 1, 2, and 3 damage states.

- **frag.m** is an algorithm to compute the range for integration of predictive density function in fragility calculation.

- **mandrift\_pga.m** is an algorithm to manipulate the drift data for fragility calculation on PGA.

- **mandrift\_psa.m** is an algorithm to manipulate the drift data for fragility calculation on spectral acceleration.

- **mandrift\_psd.m** is an algorithm to manipulate the drift data for fragility calculation on spectral displacement.

- **post\_mc.m** is an algorithm to calculate the posterior density function.

- **pred\_mc.m** is an algorithm to calculate the predictive density function.

- **fx*d** is a subroutine to change a vector to matrix or 3-dimensional matrix according to the number of damage states in the analysis. \((*\text{=}2,3)\)

- **norm\_fx*d** is a subroutine to normalize the predictive density function. \((*\text{=}1,2,3)\)
• **paramindex** is an algorithm to find the index of parameter according to damage state and its divisions.

• **mar*d** is a subroutine to calculate the marginal predictive density function. (*=2,3)

• **spectrum_generator** (written by Richard Wood at UCSD) is an algorithm to generate the spectral response for the given set of ground motions.

• **frag_wall_indep** is a subroutine to compute the fragility based on Eq.(3.1) of Section 3.

• **frag_wall_dep** is a subroutine to compute the fragility based on Eq.(3.2) of Section 3.
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</tr>
</thead>
<tbody>
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52
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